

Generation of Invariants*

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Let G be any group acting on the Weyl algebra $W_1(\mathbb{C})$. We shall propose a method for finding explicit generators of the ring of invariants $W_1(\mathbb{C})^G$. This method can be generalized to the case of the ring of invariants of an almost normalizing extension R over a commutative ring C under the linear action of a finite group G , provided that $|G|$ is invertible in C and a set of explicit generators of $\text{gr}(R)^G$ is known. © 1999 Academic Press

Key Words: Rings of invariants; Weyl algebras; enveloping algebras; almost normalizing extensions.

1. INTRODUCTION

Let K be any field, G a finite subgroup of $GL_n(K)$. Then G acts naturally on the polynomial ring $K[x_1, \dots, x_n]$. A classical result of Emmy Noether is that the ring of invariants $K[x_1, \dots, x_n]^G := \{f \in K[x_1, \dots, x_n] : \sigma(f) = f \text{ for any } \sigma \in G\}$ is finitely generated over K [N2]; moreover, when $\text{char } K = 0$, a procedure was proposed to produce such a set of generators [N1]. For further developments, see [HK].

The analogous result of the above finiteness theorem in the non-commutative case was proved by Montgomery and Small. Namely,

THEOREM 1.1 [MS, Theorem 1 and Theorem 2]. *Let C be a commutative noetherian ring. R a finitely generated C -algebra, which is possibly not commutative. Suppose that G is a finite group of C -automorphisms of R and R^G is the ring of invariants of R under the action of G . Assume that (i) R is left noetherian and $|G|^{-1} \in R$, or (ii) R is a noetherian PI integral domain and R^G is noetherian. Then R^G is also finitely generated over C .*

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We might mention another result along this line, where the role of R^G is replaced by S .

THEOREM 1.2 (Lorenz [L, Corollary 2]). *Let C be a commutative ring, $S \subset R$ be C -algebras. Assume that R is a finitely generated C -algebra, and both R and $\text{Tr}_S(R)$ are finitely generated left S -modules where $\text{Tr}_S(R)$ is the trace ideal of R in S . Then S is a finitely generated C -algebra if and only if so is $S/\text{Tr}_S(R)$.*

The motivation of this paper is to find R^G explicitly in some concrete cases, e.g., Weyl algebras and enveloping algebras. It turns out that the non-commutative ring of invariants is closely related to that of its associated graded ring, which is commutative and can be determined explicitly by [HK, Theorem 4.9]. For example, we can prove the following

THEOREM 1.3. *Let K be any field with $\text{char } K = 0$, $A_n(K)$ the n th Weyl algebra over K , $\text{gr}(A_n(K))$ the associated graded ring of $A_n(K)$. Suppose that G is a finite group of K -automorphisms acting linearly on $A_n(K)$. If $\text{gr}(A_n(K))^G = K[\bar{g}_1, \bar{g}_2, \dots, \bar{g}_m]$, where g_1, \dots, g_m are elements in $A_n(K)$, then $A_n(K)^G = K[\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_m]$, where, for $1 \leq i \leq m$, \tilde{g}_i is defined by*

$$\tilde{g}_i = \frac{1}{|G|} \sum_{\sigma \in G} \sigma(g_i).$$

(See Definition 1.5 for the definition of linear actions.)

Note that $\text{gr}(A_n(K))$ is a polynomial ring in $2n$ variables and its ring of invariants can be computed quite efficiently [N1, HK]. Thus the above theorem not only establishes the finite generation of $A_n(K)^G$, but also proposes a method for exhibiting a set of generators.

Weyl algebras and enveloping algebras are special cases of almost normalizing extensions, and the above theorems can be generalized to these non-commutative rings. So let us recall their definitions.

DEFINITION 1.4 [CR, p. 28]. Let C be any commutative ring, R a finitely generated C -algebra with generators a_1, a_2, \dots, a_r . R is called an almost normalizing extension of C if $a_i a_j - a_j a_i \in \sum_{k=1}^r C \cdot a_k + C$, for any i, j .

DEFINITION 1.5. Let G be a finite group of C -automorphisms of an almost normalizing extension $R = C[a_1, \dots, a_r]$ of C . We say that the action of G on R is linear if $\sigma(a_i) \in \sum_{j=1}^r C \cdot a_j$, for $1 \leq i \leq r$.

It is clear that, in this situation, G leaves invariant the “standard” filtration $\{F_n : 0 \leq n < \infty\}$, where $F_n :=$ the left C -module generated by $a_{i_1}, a_{i_2} \cdot a_{i_k}$ with $0 \leq k \leq n$. Hence G induces a natural action on each F_n/F_{n-1} , and thus on the associated graded ring $\text{gr}(R) := F_0 \oplus (F_1/F_2)$

$\oplus \cdots \oplus (F_n/F_{n-1}) \oplus \cdots$ also. Elements in F_n/F_{n-1} will be called homogeneous elements in $\text{gr}(R)$.

Theorem 1.3 can be generalized to the following.

THEOREM 1.6. *Let C be a commutative ring, G be a finite group of C -automorphisms acting linearly on R , an almost normalizing extension of C . Assume that $|G|$ is invertible in C . If $\text{gr}(R)^G = C[g_1, \dots, g_m]$, where each g_i is a homogeneous element in $\text{gr}(R)$, then $R^G = C[\Psi(g_1), \dots, \Psi(g_m)]$. In particular, R^G is finitely generated over C . (For the definition of $\Psi(g)$, see Definition 2.2. Note that $\text{gr}(R)^G$ is also a finitely generated C -algebra because of Theorem 1.2.)*

The proof of Theorem 1.6 will be given in Section 2. In Section 3, we shall exhibit the rings of invariants of the first Weyl algebra $A_1(\mathbb{C})$ under any finite group actions.

Standing Notations. Every ring in this paper has an identity element. It is unnecessary to assume that the ring we consider is noetherian or left noetherian. If G is a group acting on a ring R , we shall denote the ring of invariants by R^G , i.e., $R^G := \{a \in R : \sigma(a) = a \text{ for any } \sigma \in G\}$. If C is any commutative ring, the n th Weyl algebra over C , denoted by $A_n(C)$, is the C -algebra with $2n$ generators $x_1, \dots, x_n, y_1, \dots, y_n$ and relations $x_i y_i - y_i x_i = 1$ (for $1 \leq i \leq n$), $x_i y_j = y_j x_i$ (for $i \neq j$), $x_i x_j = x_j x_i$, $y_i y_j = y_j y_i$ (for any i, j). We shall emphasize that $\text{gr}(A_n(C))$ is isomorphic to the polynomial ring in $2n$ variables, $\bar{x}_1, \dots, \bar{x}_n, \bar{y}_1, \dots, \bar{y}_n$ over C ; i.e., in the standard filtration of $A_n(C)$, F_1 is chosen as $F_1 = C + \sum_{1 \leq i \leq n} Cx_i + \sum_{1 \leq i \leq n} Cy_i$. The reader can consult [CR, GW, S] for unexplained terminologies.

2. THE PROOF OF THEOREM 1.6

In this section we shall assume, unless otherwise specified, that C is any commutative ring, $R = C[a_1, \dots, a_r]$ is an almost normalizing extension of C with generators a_1, \dots, a_r , and G is a finite group of C -automorphisms acting linearly on R .

DEFINITION 2.1. If $f \in R$ can be written as $f = \sum c_\lambda a_{\lambda_1} a_{\lambda_2} \cdots a_{\lambda_n} + f_0 \in F_n$, where $f_0 \in F_{n-1}$, $c_\lambda \in C$, and $\lambda_i \in \{1, 2, \dots, r\}$, the formal degree of f is defined as n . Note that the formal degree of an element in R is not uniquely determined; it depends on the form how f is written as a (non-commutative) polynomial in a_1, \dots, a_r . In the case of the Weyl algebra $A_n(C)$, the formal degree of the element $x_1 y_1 - y_1 x_1 = 1$ can be 2 or 0.

DEFINITION 2.2. For the convenience of exposition, we shall associate a homogeneous element $\Phi(f) \in \text{gr}(R)$ to each element $f \in R$: Suppose that the formal degree of $f \in R$ is n ; then $\Phi(f)$ is the image of f in F_n/F_{n-1} .

On the other hand, if G is a finite group of C -automorphisms acting linearly on R and $g \in F_n/F_{n-1} \subset \text{gr}(R)$ is a homogeneous element, we shall associate an element $\Psi(g) \in R$ to g as follows: Choose a preimage f in F_n of g and define

$$\Psi(g) := \frac{1}{|G|} \sum_{\sigma \in G} \sigma(f) \in F_n.$$

Note that both $\Phi(f)$ and $\Psi(g)$ are not uniquely determined.

LEMMA 2.3. (1) If $f \in R^G$ and the formal degree of f is n with $\Phi(f) \in F_n/F_{n-1}$, then $\Phi(f) \in \text{gr}(R)^G$.

(2) If $g \in \text{gr}(R)^G$ and g is a homogeneous element of degree n , then $\Psi(g) \in R^G \cap F_n$.

Proof. (1) Write $f = f_1 + f_0$, where $f_0 \in F_{n-1}$ and $f_1 = \sum_{\lambda} c_{\lambda} a_{\lambda_1} \cdots a_{\lambda_n}$. Then, for any $\sigma \in G$, $f = \sigma(f) = \sigma(f_1) + \sigma(f_0)$. Hence $\sigma(f_1) - f_1 = f_0 - \sigma(f_0) \in F_{n-1}$.

(2) The proof is left to the reader.

Now we shall give the proof of Theorem 1.6.

We may assume that each g_i is not zero. It suffices to prove $R^G \cap F_n \subset C[\Psi(g_1), \dots, \Psi(g_m)]$ by induction on n .

Suppose that $f \in R^G \cap F_n$. Without loss of generality we may assume that $f \notin F_{n-1}$ and the formal degree of f is n . Thus $\Phi(f) \in (F_n/F_{n-1})^G$ by Lemma 2.3. Hence we may write $\Phi(f) = \sum_{\lambda} c_{\lambda} g_1^{\lambda_1} \cdots g_m^{\lambda_m}$, where $c_{\lambda} \in C$ and $\lambda = (\lambda_1, \dots, \lambda_m)$ runs over all non-negative m -tuples with $n = \sum_{i=1}^m \lambda_i (\deg g_i)$.

Since $f \in R^G$, it follows that $f - \Psi(\Phi(f)) \in F_{n-1}$. Thus $f - \sum_{\lambda} c_{\lambda} \Psi(g_1)^{\lambda_1} \cdots \Psi(g_m)^{\lambda_m} \in F_{n-1}$. (Note that $\sum_{\lambda} c_{\lambda} \Psi(g_1)^{\lambda_1} \cdots \Psi(g_m)^{\lambda_m}$ and $\sum_{\lambda} c_{\lambda} \Psi(g_1)^{\lambda_1} \cdots \Psi(g_m)^{\lambda_m}$ are not equal in general. But they differ by an element in F_{n-1} .) Apply the induction hypothesis.

Finally, to prove that R^G is finitely generated, it suffices to establish the finite generation of the commutative C -algebra $\text{gr}(R)^G$, which can be achieved by Theorem 1.2.

THEOREM 2.4. The notations are the same as in Theorem 1.6. Then R is a finitely generated module over R^G .

Proof. By standard arguments as Noether's proof of finite generation [N2], we can find $q_1 = 1, q_2, \dots, q_k$ such that

$$\text{gr}(R) = \sum_{j=1}^k \text{gr}(R)^G \cdot q_j,$$

where q_j is homogeneous. Choose a preimage p_j of q_j such that the formal degree of p_j is $\deg q_j$. We shall show that

$$R = \sum_{j=1}^k R^G \cdot p_j.$$

Suppose that $f \in F_n \setminus F_{n-1}$. Then $\Phi(f) \in F_n/F_{n-1}$. Write

$$\Phi(f) = \sum_{j=1}^k h_j \cdot q_j,$$

where h_j is a homogeneous element in $\text{gr}(R)^G$ and $\deg h_j + \deg q_j = n$. It follows that

$$f - \sum_{j=1}^k \Psi(h_j) \cdot p_j \in F_{n-1}.$$

Now apply the induction hypothesis to $f - \sum \Psi(h_j) \cdot p_j$.

Remark 2.5. One may wonder whether the converse of Theorem 1.6 is true, i.e., if $R^G = \mathbb{C}[f_1, \dots, f_m]$, is it necessary that $\text{gr}(R)^G = \mathbb{C}[\Phi(f_1), \dots, \Phi(f_m)]$, where $\Phi(f_i)$ is defined by choosing the minimal formal degree of f_i ? The negative answer to this question and the example described below were kindly communicated to us by the referee.

Let $R = A_1(\mathbb{C})$ be the complex Weyl algebra with generators x and y and relation $xy - yx = 1$. Define a \mathbb{C} -automorphism $\sigma: R \rightarrow R$ by $\sigma(x) = -x$, $\sigma(y) = -y$. Then $R^{\langle \sigma \rangle} = \mathbb{C}[x^2, xy, y^2] = \mathbb{C}[x^2, y^2]$ because $x^2y^2 - y^2x^2 = 4xy - 2$. On the other hand, $\text{gr}(R) \simeq \mathbb{C}[\bar{x}, \bar{y}]$ is the polynomial ring in two variables \bar{x} and \bar{y} over \mathbb{C} , and the induced automorphism is defined by $\sigma(\bar{x}) = -\bar{x}$, $\sigma(\bar{y}) = -\bar{y}$. Note that $\mathbb{C}[\bar{x}, \bar{y}]^{\langle \sigma \rangle} = \mathbb{C}[\bar{x}^2, \bar{xy}, \bar{y}^2]$ and it can never be generated by two elements over \mathbb{C} . Otherwise, $\mathbb{C}[\bar{x}, \bar{y}]^{\langle \sigma \rangle}$ would be a polynomial ring and it would be necessary that σ should be a pseudo-reflection by the Chevalley–Shephard–Todd theorem [S, Theorem 4.2.5, p. 76].

3. EXPLICIT GENERATORS OF $A_1(\mathbb{C})^G$

We shall apply Theorem 1.6 to compute explicitly the generators of $A_1(\mathbb{C})^G$ where G is a finite group of \mathbb{C} -automorphisms acting on the first Weyl algebra $A_1(\mathbb{C})$ over the complex number field \mathbb{C} .

Recall that $A_1(\mathbb{C})$ is generated by x and y over \mathbb{C} with the relation $xy - yx = 1$. We shall denote by $\mathbb{C}[x, y]$ the (commutative) polynomial ring in two variables over \mathbb{C} .

It is Alev who proves that every finite group G of \mathbb{C} -automorphisms acting on $A_1(\mathbb{C})$ is linearizable [A]. Moreover, the finite linear subgroups acting on $A_1(\mathbb{C})$ are just the finite subgroups in $SL_2(\mathbb{C})$ [AD, 2.1.]. These subgroups are cyclic groups, binary dihedral groups, the binary tetrahedral group, the binary octahedral group, and the binary icosahedral group [S, pp. 88–94].

To compute $A_1(\mathbb{C})^G$, we compute first the invariants $\mathbb{C}[x, y]^G$ in the commutative case following the results of [S, pp. 94–97] and using MAPLE. We then apply Theorem 1.6 to modify generators in the commutative cases to get those for $A_1(\mathbb{C})^G$. This time it is necessary to write a small computer program so that generators for $A_1(\mathbb{C})^G$ will be expressed in some “standard” forms. We list our result as follows.

THEOREM 3.1. *Let $\zeta_n = \exp(2\pi\sqrt{-1}/n) \in \mathbb{C}$ and*

$$\begin{aligned}\theta_n &= \begin{pmatrix} \zeta_n & 0 \\ 0 & \zeta_n^{-1} \end{pmatrix}, & \mu &= \begin{pmatrix} 0 & \sqrt{-1} \\ \sqrt{-1} & 0 \end{pmatrix}, & \nu &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\ \eta &= \frac{1}{\sqrt{2}} \begin{pmatrix} \zeta_8^7 & \zeta_8^7 \\ \zeta_8^5 & \zeta_8 \end{pmatrix}, & \varphi &= \begin{pmatrix} -\zeta_5^3 & 0 \\ 0 & -\zeta_5^2 \end{pmatrix}, \\ \psi &= \frac{1}{\zeta_5^2 - \zeta_5^{-2}} \begin{pmatrix} \zeta_5 + \zeta_5^{-1} & 1 \\ 1 & -(\zeta_5 + \zeta_5^{-1}) \end{pmatrix}.\end{aligned}$$

Case 1. The cyclic group of order n : $G = \langle \theta_n \rangle$.

$$\mathbb{C}[x, y]^G = \mathbb{C}[x^n, y^n, xy].$$

Generators of $A_1(\mathbb{C})^G$: x^n, y^n, xy .

Case 2. The binary dihedral group of order $4n$: $G = \langle \theta_{2n}, \mu \rangle$.

$$\mathbb{C}[x, y]^G = \mathbb{C}[x^2y^2, x^{2n} + (-1)^n y^{2n}, x^{2n+1}y - (-1)^n xy^{2n+1}].$$

Generators of $A_1(\mathbb{C})^G$:

$$y^2x^2 + 2yx,$$

$$x^{2n} + (-1)^n y^{2n},$$

$$2\{yx^{2n+1} - (-1)^n y^{2n+1}x\} + (2n+1)\{x^{2n} - (-1)^n y^{2n}\}.$$

Case 3. The binary tetrahedral group of order 24: $G = \langle \theta_4, \mu, \eta \rangle$.

$$\begin{aligned}\mathbb{C}[x, y]^G = \mathbb{C}[xy^5 - x^5y, x^8 + 14x^4y^4 + y^8, \\ x^{12} - 33x^8y^4 - 33x^4y^8 + y^{12}].\end{aligned}$$

Generators of $A_1(\mathbb{C})^G$:

$$2y^5x - 2yx^5 + 5y^4 - 5x^4,$$

$$y^8 + 14y^4x^4 + x^8 + 112y^3x^3 + 252y^2x^2 + 168yx,$$

$$\begin{aligned}y^{12} - 33y^8x^4 - 33y^4x^8 + x^{12} - 528y^7x^3 - 528y^3x^7 - 2772y^6x^2 \\ - 2772y^2x^6 - 5544y^5x - 5544yx^5 - 3465y^4 - 3465x^4.\end{aligned}$$

Case 4. The binary octahedral group of order 48: $G = \langle \theta_8, \mu, \eta \rangle$.

$$\begin{aligned}\mathbb{C}[x, y]^G = \mathbb{C}[x^8 + 14x^4y^4 + y^8, x^{10}y^2 - 2x^6y^6 + x^2y^{10}, \\ x^{17}y - 34x^{13}y^5 + 34x^5y^{13} - xy^{17}],\end{aligned}$$

Generators of $A_1(\mathbb{C})^G$:

$$y^8 + 14y^4x^4 + x^8 + 112y^3x^3 + 252y^2x^2 + 168yx,$$

$$\begin{aligned}y^{10}x^2 - 2y^6x^6 + y^2x^{10} + 10y^9x - 36y^5x^5 + 10yx^9 \\ + \frac{45}{2}y^8 - 225y^4x^4 + \frac{45}{2}x^8 - 600y^3x^3 - 675y^2x^2 - 270yx,\end{aligned}$$

$$\begin{aligned}y^{17}x - 34y^{13}x^5 + 34y^5x^{13} - yx^{17} + \frac{17}{2}y^{16} - 1105y^{12}x^4 \\ + 1105y^4x^{12} - \frac{17}{2}x^{16} - 13260y^{11}x^3 + 13260y^3x^{11} - 72930y^{10}x^2 \\ + 72930y^2x^{10} - 182325y^9x + 182325yx^9 + \frac{328185}{2}y^8 + \frac{328185}{2}x^8.\end{aligned}$$

Case 5. The binary icosahedral group of order 120: $G = \langle \varphi, \nu, \psi \rangle$.

$$\begin{aligned}\mathbb{C}[x, y]^G = \mathbb{C}[x^{11}y + 11x^6y^6 - xy^{11}, x^{20} - 228x^{15}y^5 + 494x^{10}y^{10} \\ + 228x^5y^{15} + y^{20}, x^{30} + 522x^{25}y^5 - 10005x^{20}y^{10} \\ - 10005x^{10}y^{20} - 522x^5y^{25} + y^{30}].\end{aligned}$$

Generators of $A_1(\mathbb{C})^G$:

$$\begin{aligned}
& y^{11}x - 11y^6x^6 - yx^{11} + \frac{11}{2}y^{10} - 198y^5x^5 - \frac{11}{2}x^{10} \\
& - \frac{2475}{2}y^4x^4 - 3300y^3x^3 - \frac{7425}{2}y^2x^2 - 1485yx, \\
& y^{20} + 228y^{15}x^5 + 494y^{10}x^{10} - 228y^5x^{15} + x^{20} \\
& + 8550y^{14}x^4 + 24700y^9x^9 - 8550y^4x^{14} + 119700y^{13}x^3 \\
& + 500175y^8x^8 - 119700y^3x^{13} + 778050y^{12}x^2 \\
& + 5335200y^7x^7 - 778050y^2x^{12} + 2334150y^{11}x + 32678100y^6x^6 \\
& - 2334150yx^{11} + 2567565y^{10} + 117641160y^5x^5 - 2567565x^{10} \\
& + 245085750y^4x^4 + 280098000y^3x^3 + 157555125y^2x^2 \\
& + 35012250yx, \\
& y^{30} - 522y^{25}x^5 - 10005y^{20}x^{10} - 10005y^{10}x^{20} + 522y^5x^{25} + x^{30} \\
& - 32625y^{24}x^4 - 1000500y^{19}x^9 - 1000500y^9x^{19} + 32625y^4x^{24} \\
& - 783000y^{23}x^3 - 42771375y^{18}x^8 - 42771375y^8x^{18} + 783000y^3x^{23} \\
& - 9004500y^{22}x^2 - 1026513000y^{17}x^7 - 1026513000y^7x^{17} \\
& + 9004500y^2x^{22} - 49524750y^{21}x - 15269380875y^{16}x^6 \\
& - 15269380875y^6x^{16} + 49524750yx^{21} - 104001975y^{20} \\
& - 146586056400y^{15}x^5 - 146586056400y^5x^{15} + 104001975x^{20} \\
& - 916162852500y^{14}x^4 - 916162852500y^4x^{14} - 3664651410000y^{13}x^3 \\
& - 3664651410000y^3x^{13} - 8932587811875y^{12}x^2 - 8932587811875y^2x^{12} \\
& - 11910117082500y^{11}x - 11910117082500yx^{11} - 6550564395375y^{10} \\
& - 6550564395375x^{10}.
\end{aligned}$$

Proof. To compute the generators of $A_1(\mathbb{C})^G$, we should take care of the non-commutativity. In particular, it is necessary to write $y^m x^n \cdot y^p x^q$ in terms of the linear combination of “monomials” $y^a x^b$. More precisely, from the formula

$$y^m x^n \cdot y^p x^q = \sum_{i=0}^{\min(n, p)} \binom{n}{i} \frac{p!}{(p-i)!} y^{m+p-i} x^{n+q-i},$$

use the following procedure in MAPLE V language:

```

proc := proc(sd, td)
local sn, tn, sm, tm, m, n, p, q, i, c, pe;
  sn := indets (sd);
  tn := indets (td);
  pe := 0;
  for sm to nops (sn) do
    m := op(sn[sm])[1];
    n := op(sn[sm])[2];
    for tm to nops (tn) do
      p := op (tn[tm])[1];
      q := op (tn[tm])[2];
      for i from 0 to min(n, p) do
        c := binomial (n, i) * p! / (p - i)!;
        c := c * coeff(sd, X[m, n]) * coeff(td, X[p, q]);
        pe := pe + c * X[m + p - i, n + q - i]
      od
    od;
  pe
end

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where $X[m, n]$ denotes $y^m x^n$. The rest is left to the reader.

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